This is a notice concerning the September 2008 "DSP Tips & Tricks" column article (pages 113–119) titled: "Precise Variable-Q Filter Design" by Shlomo Engelberg. There is a simplification to the article's Equation (5). Here is the story.

From References [1]–[3], the standard lowpass to lowpass digital filter transformation process defines Eq. (5)'s parameter *a* to be:

$$a = \frac{\tan(\pi F_O / F_S - \pi F_D / F_S)}{\sin(2\pi F_D / F_S) + \cos(2\pi F_D / F_S)\tan(\pi F_D / F_S - \pi F_D / F_S)}$$

where F_o is the critical frequency of the original filter, F_D is the desired critical frequency of the transformed filter, and F_S is the filters' sample rate. Dr. Engelberg's 'trick' is to set $F_o = F_S/4$, yielding the article's printed expression for parameter *a* of:

$$a = \frac{\tan(\pi / 4 - \pi F_D / F_S)}{\sin(2\pi F_D / F_S) + \cos(2\pi F_D / F_S) \cdot \tan(\pi / 4 - \pi F_D / F_S)}. \qquad (5)$$

In September 2008, our Signal Processing magazine's Associate Editor Dr. Alen Docef discovered that the *messy* denominator of Eq. (5) was always equal to one (unity). So the simplified Eq. (5) becomes:

$$a = \tan(\pi / 4 - \pi F_D / F_S). \qquad (5\text{-new})$$

Dr. Docef's discovery was that the denominator of the original Eq. (5) can be expressed in the form:

$$\sin(2x) + \cos(2x) \cdot \tan(\pi/4 - x) = \frac{\sin(2x) \cdot \cos(\pi / 4 - x) + \cos(2x) \cdot \sin(\pi / 4 - x)}{\cos(\pi / 4 - x)}. \qquad (I)$$

Realizing that $\sin(\alpha)\cdot\cos(\beta) + \cos(\alpha)\cdot\sin(\beta) = \sin(\alpha + \beta)$, and $\sin(\theta) = \cos(\pi/2 - \theta)$, Eq. (I) becomes:

$$\frac{\sin(2x + \pi / 4 - x)}{\cos(\pi / 4 - x)} = \frac{\sin(\pi / 4 + x)}{\cos(\pi / 4 - x)} = \frac{\cos(\pi / 4 - x)}{\cos(\pi / 4 - x)} = 1 \qquad (II)$$

Because Eq. (II) is the denominator of the article's Eq. (5), we obtain the above simplified Eq. (5-new).

—Rick Lyons [Oct. 5, 2008]